The Problem of Selection of a Set of Partially Distinguishable Guards

Anna Gorbenko
Department of Intelligent Systems and Robotics
Ural Federal University
620083 Ekaterinburg, Russia
gorbenko.ann@gmail.com

Vladimir Popov
Department of Intelligent Systems and Robotics
Ural Federal University
620083 Ekaterinburg, Russia
Vladimir.Popov@usu.ru

Abstract

In this paper we consider the problem of selection of a set of partially distinguishable guards. We describe an approach to solve the problem. This approach is based on an explicit reduction from the problem to the satisfiability problem.

Keywords: partially distinguishable guards, satisfiability, NP-complete

Visual landmarks problems has been extensively studied in robotics (see e.g. [1] – [3]). In particular, the following problem was proposed in [4].

Given a polygon $P$ and a finite set of candidate guard locations $N \subset P$, can one efficiently choose the guard set $S \subseteq N$ that minimizes the number of colors required?

A point $p \in P$ is visible from point $q \in P$ if the closed segment $[p, q]$ is a subset of $P$. Let $p \leftrightarrow q$ if and only if $p$ is visible from $q$. The visibility polygon $Vis(p)$ of a point $p \in P$ is defined as

$$Vis(p) = \{q \in P \mid p \leftrightarrow q\}.$$ 

Since $N$ is a finite set, we can find $Vis(p)$, for any $p \in N$. Also, in view of finiteness of $N$, we can consider $P$ as a finite part of two-dimensional integer
A. Gorbenko and V. Popov

Therefore, we can consider the following decision version of the problem from [4].

**The problem of selection of a set of partially distinguishable guards (SG):**

**Instance:** A grid graph \( P = (V, E) \), a finite subset \( N \) of the set of vertices of \( P \), \( Vis(p) \), for any \( p \in N \), and positive integer \( k \).

**Question:** Are there a set \( S \subseteq N \) and function \( C : S \rightarrow \{1, \ldots, k\} \)

such that

\[ V = \bigcup_{p \in S} Vis(p) \]

and for any \( p, q \in S \), if \( C(p) = C(q) \), then \( q \notin Vis(p) \)?

Note that SG is \( \text{NP} \)-complete [4]. Encoding hard problems as instances of different variants of the satisfiability problem and solving them with very efficient satisfiability algorithms has caused considerable interest (see e.g. [5] – [18]). We consider an explicit reduction from SG to the satisfiability problem.

Let \( P = \{a_1, \ldots, a_{|P|}\}, N = \{a_{t_1}, \ldots, a_{t_{|N|}}\}, \)

\[
\begin{align*}
\varphi[1] &= \land_{1 \leq i \leq |N|} \land_{1 \leq j \leq k} x[i, j], \\
\varphi[2] &= \land_{1 \leq i \leq |N|} \land_{1 \leq j[1] < j[2] \leq k} (\neg x[i, j[1]] \lor \neg x[i, j[2]]), \\
\varphi[3] &= \land_{1 \leq i[1] < i[2] \leq |N|, \ a_{i[1]} \in Vis(a_{i[2]})} \land_{1 \leq j \leq k} (\neg w[i[1]] \lor \neg w[i[2]] \lor \neg x[i, j[1]] \lor \neg x[i, j[2]]), \\
\varphi[4] &= \land_{1 \leq i \leq |V|} \land_{1 \leq j \leq |N|} y[i, j], \\
\varphi[5] &= \land_{1 \leq i \leq |V|} \land_{1 \leq j[1] < j[2] \leq |N|} (\neg y[i, j[1]] \lor \neg y[i, j[2]]), \\
\varphi[6] &= \land_{1 \leq i \leq |V|, a_{i} \in Vis(a_{j})} \land_{1 \leq j \leq |N|} (\neg y[i, j] \lor w[j]), \\
\xi &= \land_{i=1}^{6} \varphi[i].
\end{align*}
\]

It is easy to check that there are a set \( S \subseteq N \) and function \( C : S \rightarrow \{1, \ldots, k\} \)
such that \( V = \bigcup_{p \in S} Vis(p) \) and for any \( p, q \in S \), if \( C(p) = C(q) \), then \( q \notin Vis(p) \) if and only if \( \xi \) is satisfiable. Clearly, \( \xi \) is a CNF. So, \( \xi \) gives us an explicit reduction from SG to SAT. Using standard transformations (see e.g. [19]) we can obtain an explicit transformation \( \xi \) into \( \zeta \) such that \( \xi \iff \zeta \) and \( \zeta \) is a 3-CNFSAT. It is easy to see that \( \zeta \) gives us an explicit reduction from SG to 3SAT.

We have designed a generator of natural instances for the problem SG. We consider our genetic algorithms OA[1] (see [20]), OA[2] (see [21]), OA[3] (see
Table 1: Experimental results for SG.

<table>
<thead>
<tr>
<th></th>
<th>time</th>
<th>average</th>
<th>max</th>
<th>best</th>
</tr>
</thead>
<tbody>
<tr>
<td>OA[1]</td>
<td>4.2 h</td>
<td>13.85 h</td>
<td>17.3 min</td>
<td></td>
</tr>
<tr>
<td>OA[2]</td>
<td>2.93 h</td>
<td>11.37 h</td>
<td>19.23 min</td>
<td></td>
</tr>
<tr>
<td>OA[3]</td>
<td>3.67 h</td>
<td>18.41 h</td>
<td>21.41 min</td>
<td></td>
</tr>
<tr>
<td>OA[4]</td>
<td>3.84 h</td>
<td>17.2 h</td>
<td>18.6 min</td>
<td></td>
</tr>
</tbody>
</table>

[22]), and OA[4] (see [23]) for SAT. We used heterogeneous cluster. Each test was runned on a cluster of at least 100 nodes. Selected experimental results are given in Table 1.

ACKNOWLEDGEMENTS. The work was partially supported by Analytical Departmental Program “Developing the scientific potential of high school” 8.1616.2011.

References


Received: November 1, 2012