Mathematical Model of the Local Stability of the Enterprise to its Vendors

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Abstract

The paper presents an elementary model of interaction between the enterprise with suppliers. Ranked dependence of business continuity of supply disruptions. Is an amendment to the inherent stability of the enterprise.

Keywords: supplier selection, supply chain management, risk management, supplier interaction, weighted sum.

1 Introduction

The method of calculation the \(K_j^{out}\) factor is proposed and substantiated below [1-4], where \(K_j^{out}\) is the instability factor of outside environment of the network model \(G=(N, A)\) node \(j\) using theory. Research in dependence on suppliers carried out for example in [5-7].

Let’s fix the node-enterprise \(P_0\) we are interested in and the external stability factor that we would like to calculate. The distance between a pair of vertices in the \(G\) network is the magnitude of the shortest path connecting this pair of vertices. It is considered that hereinafter the first index \(i\) in the \(P_{ij}\) node designation is a distance from the node \(P_0\), and the second index \(j\) is an ordinal number of an enterprise located at a distance \(i\) from the node \(P_0\).

Let the node \(P_0\) of the network model has \(N\) adjacent vertices \(P_{11}, P_{12}, \ldots, P_{1N}\), i.e. nodes located at a distance 1 from the node \(P_0\).
Then \( P_{11}, P_{12}, \ldots, P_{1N} \) enterprises make direct surrounding of the corporate network \( P_0 \) element, i.e. they are its suppliers.

2 The method of calculation the instability factor of a corporate network node

According to the statistical monitoring during a long period of time let’s consider that statistical probabilities of failure the corresponding node \( \bar{p}_{11}, \bar{p}_{12}, \ldots, \bar{p}_{1N} \) are attributed to each of the nodes \( P_{11}, P_{12}, \ldots, P_{1N} \). For instance, failure to meet the contractual commitments is priority instability factors of respective nodes. These priority instability coefficients of external nodes have to be integral statistical characteristics:

\[
\bar{p}_{ij} = \frac{1}{t_f - t_0} \int_{t_0}^{t_f} S_j(t) v_j(t) dt,
\]

where \( t_f - t_0 \) - the statistical monitoring time interval;

\( S_j(t) \) - the distribution function (density) of failures by \( j \) enterprise for time;

\( v_j(t), 0 \leq v_j(t) \leq 1 \) - the weight function characterizing, for instance, an extent of product importance at a given time \( t \) supplied from a \( j \) enterprise (in fact, \( v_j(t) \) is strength of links (attraction force) between the nodes \( P_0 \) and \( P_{1j} \)).

It is obvious that the formula gives a dimensionless quantity because the integrand gives a dimension of time.

Simplified discrete statistical analogue looks like this:

\[
\bar{p}_{1j} = \frac{N_j^{\text{fail}}}{N_j^{\text{all}}} \cdot v_j
\]

where \( N_j^{\text{all}} \) - the total number of contracts concluded with the \( P_{1j} \) node during the observed period of time;

\( N_j^{\text{fail}} \) - the number of failures, breakdowns and various failures to comply contractual commitments during the same period of time;

\( v_j \) - the weight coefficient \( 0 \leq v_j \leq 1 \) characterizing strength of links (attraction force) between the network nodes.

Instability factors of supplier-nodes \( \bar{p}_{11}, \bar{p}_{12}, \ldots, \bar{p}_{1N} \) will be considered as the probabilities of failure the contractual commitments and the external environment instability factor \( K_0^{\text{out}} \) will be calculated as a probability of failure for this amount of enterprises adjacent to the node \( P_0 \) that the total amount of
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supplies from the allied enterprises is less than the total demand of the node $P_0$.
Let’s consider a following case: The venture $P_0$ needs external supplies only for a one type of product which volume is $A^m$. The Table 1 presents all available data for all enterprises $P_{1j}$, $1 \leq j \leq N$, supplying production.

<table>
<thead>
<tr>
<th>Nodes</th>
<th>$P_{11}$</th>
<th>$P_{12}$</th>
<th>...</th>
<th>$P_{1j}$</th>
<th>...</th>
<th>$P_{1N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stability factors</td>
<td>$p_{11}$</td>
<td>$p_{12}$</td>
<td>...</td>
<td>$p_{1j}$</td>
<td>...</td>
<td>$p_{1N}$</td>
</tr>
<tr>
<td>Volume of supplies</td>
<td>$V_{11}$</td>
<td>$V_{12}$</td>
<td>...</td>
<td>$V_{1j}$</td>
<td>...</td>
<td>$V_{1N}$</td>
</tr>
<tr>
<td>against the contract</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Possible supply increase (reserve)</td>
<td>$\Delta_{11}$</td>
<td>$\Delta_{12}$</td>
<td>...</td>
<td>$\Delta_{1j}$</td>
<td>...</td>
<td>$\Delta_{1N}$</td>
</tr>
<tr>
<td>Extent of importance (attraction force)</td>
<td>$\nu_{11}$</td>
<td>$\nu_{12}$</td>
<td>...</td>
<td>$\nu_{1j}$</td>
<td>...</td>
<td>$\nu_{1N}$</td>
</tr>
</tbody>
</table>

Table 1. The baseline data organization.

It is assumed that strength of links between the node $P_0$ and all nodes supplying the considered type of production is equal, i.e. all quantities $\nu_{1j}$ are equal. However, the possibility of different weight coefficients $\nu_{1j}$ for the one-profile partners is not excluded.

Probability space is made up of all kinds of $N$:

$$(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N), \text{ where } \varepsilon_j = \begin{cases} 0, & \text{if the node } P_{1j} \text{ has failed} \\ 1, & \text{if the node } P_{1j} \text{ has not failed} \end{cases}$$

i.e. an elementary event reflects the situation of performance and non-performance the contract per the reporting period by each of $N$ partners supplying production.

According to the Bernoulli scheme the probability of fixed $N$ is:

$$P(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N) = \prod_{j=1}^{N} p_{1j}^{\varepsilon_j}$$

Where $p_{1j}^{\varepsilon_j} = \begin{cases} p_{1j}, & \text{if } \varepsilon_j = 1, \text{ then the node } j \text{ has not failed} \\ 1 - p_{1j} = \bar{p}_{1j}, & \text{if } \varepsilon_j = 0, \text{ then the node } j \text{ has failed} \end{cases}$

The Bernoulli scheme is applied to the calculation the probability of occurrence a fixed industrial situation because it is believed that supplier-nodes are independent from each other, i.e. a failure of one supplier-node does not entail a failure of the other.

It is proposed to calculate the influence of occurred elementary event to the node $P_0$ considering the strength of links coefficients $\nu_{1j}$ by the following way:

$$P(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N) = \prod_{j=1}^{N} (p_{1j}^{\varepsilon_j} \cdot \nu_{1j}^{\varepsilon_j})$$

Where $\nu_{1j}^{\varepsilon_j} = \begin{cases} 1, & \text{if } \varepsilon_j = 1, \text{ then the node } j \text{ has not failed} \\ \nu_{1j}, & \text{if } \varepsilon_j = 0, \text{ then the node } j \text{ has failed} \end{cases}$

The proposed method of calculation the particular elementary event influence to the node $P_0$ is effortless from a practical point of view because there is a low strength of link with a failed node relieves negative consequences of this
failure and steady work of a partner is extremely high evaluated: \( v_{ij}^e = 1 \).

An extremely important aspect in the proposed method of external stability factor calculation is consideration of the extent of fulfillment a contract by an enterprise-partner side in the case of failure. To take into account this fact it is proposed to introduce contract fulfillment coefficients \( \xi_{ij}(t) \) by the node \( P_{ij} \). It is obvious that the coefficient \( \xi_{ij}(t), \ 0 \leq \xi_{ij}(t) < 1 \) is a random variable that can be different in different periods of time \( t \). Undoubtedly, a \( \xi_{ij}(t) \) random variable distribution function is a value defined by means of statistical monitoring.

An elementary event \( N \) \( (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N) \) will be named as critical if the sum of supplies by supplier-nodes side is less than the total demand of the node \( P_0 \):

\[
\sum_{\varepsilon_i=1}^{N} (V_{ij} + \Delta_{ij}) + \sum_{\varepsilon_i=0}^{N} (V_{ij} \cdot \xi_{ij}(t) + 0 \cdot \Delta_{ij}) < A^m
\]

The volume of supplies by enterprises that have committed a failure in fulfilling its contractual commitments is calculated in the second sum, therefore \( \Delta_{ij} \) is multiplied by zero.

Appearances of various \( N \) \( (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N) \) are independent events hence the required external environment instability factor is:

\[
K_0^{\text{out}}(t) = \sum_{\text{critical } N} \left( \prod_{j=1}^{N} \left( p_{ij}^{e_j} \cdot \nu_{ij}^e \right) \right)
\]

Where the sum is taken over all possible critical \( N \). It is emphasized one more time that the coefficient \( K_0^{\text{out}} \) is a probability of appearance a critical situation with supplies of \( A^m \) production.

The formula to the node \( P_0 \) external stability factor is:

\[
p_0^{\text{out}}(t) = 1 - \sum_{\text{critical } N} \left( \prod_{j=1}^{N} \left( p_{ij}^{e_j} \cdot \nu_{ij}^e \right) \right)
\]

Analysis of this formula leads to conclusion that the stability of the node that we are interested in monotonically depends on the external environment stability and it is the higher the higher the integrated stability factor of the external environment is.

It is clear that the external stability coefficient \( p_0^{\text{out}}(t) \) of the node \( P_0 \) which is received every time by means of carrying out numerical experiment is a random variable because random variables are contract performance coefficients \( \xi_{ij}(t) \) by the adjacent nodes involved in calculation of \( p_0^{\text{out}}(t) \). Therefore the mathematical expectation of a random variable \( p_0^{\text{out}}(t) \) has to be taken as a specific numerical value of the coefficient \( p_0^{\text{out}}(t) \).  

3 Distribution functions of coefficients of fulfillment contracts of the corporate network supplier-nodes

On the basis of empirical observations, facts and on the assumption of
graphs appearance of the considered dependencies let’s assume in the further theoretical researches that the function approximating the frequency allocation principle \( v(x) \) is presented in exponential form:

\[
v(x) = p_{1j} \cdot e^{\alpha \left(1 - \frac{x}{x_j}\right)} = p_{1j} \cdot e^{\alpha} \cdot e^{-\frac{x}{x_j}}
\]

or

\[
v(x) = p_{1j} \cdot \exp \left( a - \frac{a}{x} \right)
\]

Where \( x \in (0; 1] \) can vary from 0 to 100% in increments of \( h \).

Undoubtedly, \( v(1) = v(100\%) = p_{1j}, v(0) = \lim_{x \to 0^+} v(x) = 0 \).

The function defined this way \( v(x) \) decreases exponentially rapidly to the left from its maximum value \( v(1) = p_{1j} \), parameter \( a \) determines the rate of decrease. The value \( v_0 \) of a frequency \( v(x) \) at a point \( x = 1 - h \) known on statistical data is used to define a general form of parameter \( a \), i.e. \( v_0 = v(1 - h) \).

It corresponds to the frequency of fulfillment a contract by 95% and more (but less than 100%) on contract terms in increments of 5%. Then \( v_0 = p_{1j} \cdot e^{-\frac{a}{1-h+a}} \) and hence \( a = \frac{h-1}{h} \cdot \ln \left( \frac{v_0}{p_{1j}} \right) \).

References


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