

# Probabilities of Diagonal and Non-Diagonal Couplings between $d$ Electrons in Transition Metal

## I. The $d$ -Band Energy

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### Abstract

It is shown that the full account of the non-diagonal couplings between  $d$  electrons sited on different atoms in a transition metal implemented within the framework of the Wills-Harrison model leads to vanishing the  $d$ -band contribution to the internal energy.

**Keywords:** Transition metal, Wills-Harrison model,  $d$ -state coupling

In the Wills-Harrison (WH) model [1] for the transition-metal internal energy, the  $d$ -band energy,  $E_b$ , is represented as follows (hereafter, per atom):

$$E_b = -\frac{1}{2} z_d \left( \frac{10 - z_d}{10} \right) W, \quad (1)$$

where  $z_d$  is the effective  $d$ -electron valence,  $W$  -  $d$ -band width:

$$W = \left( \frac{12}{N} \sum_{m=1}^N \sum_{\substack{l=1 \\ l \neq m}}^N V_d^2(\vec{r}_{ml}) \right)^{1/2}, \quad (2)$$

where  $N$  is the number of atoms,  $V_b(r)$  - effective potential of the  $d$ - $d$  interaction (hereafter, in atomic units):

$$V_b(r) = \frac{r_d^3}{r^5} K_b. \quad (3)$$

Here,  $r_d$  is the  $d$ -state radius,  $K_b$  - combinatoric coefficient, which in the WH approximation depends on diagonal only couplings between  $d$  electrons sited on different atoms:

$$K_b^{\text{WH}} = \left( \sum_{m=-2}^2 \frac{y_m^2}{5} \right)^{\frac{1}{2}}, \quad (4)$$

where  $m$  is the magnet quantum number,

$$y_m = y_{|m|} = -\frac{(-1)^{|m|} 180}{\pi(2+|m|)!(2-|m|)!}. \quad (5)$$

From (4), (5)

$$K_b^{\text{WH}} = 28.06 / \pi. \quad (6)$$

In [2] was introduced the probability  $p$  that all 25  $d$ - $d$  couplings between two different atoms in a metal are equiprobable. Then, the probability of the WH limit case that only 5 equiprobable diagonal couplings are possible is  $(1-p)$ . From this assumption, the probability of a non-diagonal coupling is  $0.8p$ , probability of a diagonal coupling is  $(1-0.8p)$  and

$$K_b = \left[ \frac{1}{5} \left( \left( 1 - \frac{4p}{5} \right) y_0^2 + \left( 2 - \frac{6p}{5} \right) (y_2^2 + y_1^2) + \frac{4p}{5} y_0(y_1 + y_2) + \frac{8p}{5} y_1 y_2 \right) \right]^{\frac{1}{2}}. \quad (7)$$

Now, allow us to apply (5) to (7). As a result,

$$K_b = K_b^{\text{WH}} \sqrt{1-p}. \quad (8)$$

This surprising result denotes that at full account of the non-diagonal couplings between  $d$  electrons sited on different atoms ( $p=1$ ), the  $d$ -band energy in a transition metal is being become equal to zero.

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### References

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