The deviation of light in a moving medium.

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In the preceding work*) it was given the deduction of law of the refraction of light in a medium, which was mowing in the direction of a normal of medium's surface. In the present work has been considered the case, when is moving in the tangential direction.

Chiefly an attention has been applied on the deviation of the ray of light by a moving medium that appears as a consequence of the law of refraction, but not on the deduction of law, which are supposed to be invastigated in one of the succeeding works upon the common case.

It succeded to indicate that in the expression for the tangent of angel of the deviation comes in Fresnel's coefficient.

$$\text{tg}\ \alpha = \beta_1 \left(1 - \frac{1}{n^2}\right), \text{ or } \frac{v\left(1 - \frac{1}{n^2}\right)}{q^1} \qquad (a)$$

The deduction founded upon the principle of relativity and the formulae of Lorenz's transformation. The authors of the present work for convenience sake of the reasoning accept Lorenz's views on the real contradiction of bodies in the direction of their motion. Let us imagine the following material arrangement and let this arrangement be at rest (fig. 1). Let the ray of light from the point A_1^1 which is on the material circle fall at the point 0 of medium II limited by a plan surface DD_1 perpendicular to the flat of plan. Let e medium I be a vacuum, let a normal to the medium in the point 0 be CC_1 the angel of incidence be α_1^1, the angel of refraction be α_2^1 and the radius of circle be r. Then we shall have

$$\sin \alpha_1^1 = n \sin \alpha_2^1 \text{ or } a_1^1, = na_2^1$$

Let us suppose now that this arrangement is mowing together with the medium II with the constant velocity v in the tangential direction DD_1 parallel to the flat of the incidence of ray. According to Lorenz's hypothesis of the contradiction this material circle shall be converted into ellipse, the equetion of which is

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*) Известия Уральского Государственного Университета вып. II, 1921 г.

$$\frac{x^2}{r^2} + \frac{y^2}{k^2 r^2} = 1, \text{ where } k = \sqrt{1 - \beta^2}, \beta = \frac{v}{c};$$

c is the velocity of light in a vaccuum. Let the ray come out from this material point A_1^1 of circle or the point A_1 in the fig. 2 and overtakes the point 0 in the position 0_1, the angel or incidence shall be already α_1. Passing farther in the medium if the ray must come to the material point A_2^1 — A_2 in the fig. 2, when 0 will get to 0_2, the angel oft refraction will be α_2.

Let us lead in the following denotations:
$A_1 0 = l_1$; $A_2 0 = l_2$; $A_1 B_1 = b_1$; $A_2 B_2 = b_2$; $A_1 C_1 = a_1$; $A_2 C_2 = a_2$; $0\,0_1 = S_1$; $0\,0_2 = S_2$ and let us calculate $\sin \alpha_1$ accordingly to the motion of ray in the vacuum. From the fig. 2: $\sin \alpha_1 = \frac{a_1 + s_1}{l_1}$

$$\text{where } l_1 = b_1^2 + (a_1 + s_1)^2 \qquad (1)$$

$$\frac{s_1}{l_1} = \frac{v}{c} = \beta$$

since s_1 and l_1 are the ways past away by the medium and ray in the same time. From the equation of ellipse

$$a_1 = k \sqrt{r^2 - b_1^2}$$

Substituting the signification s_1 in the equation (1) we get the square equation for l_1

$$l_1^2 . k^2 - 2 \beta a_1 l_1 - (a_1^2 + b_1^2) = 0$$

$$\text{hence } l_1 = \frac{\beta a_1 \pm \sqrt{a_1^2 + k^2 b_1^2};}{k^2}$$

Substituting a_1 we obtain

$$l_1 = \frac{\beta a_1 \pm k^2}{k^2} = \frac{\beta \sqrt{r^2 - b_1^2} \pm r}{k};$$

if $v = o$ ($\beta = o$) we have $l_1 = \pm r$ and since we admit that the way of ray l is positive we must take the sign of plus. Substituting l_1 in the equation $\sin \alpha_1$, we get

$$\sin \alpha_1 = \frac{k \sqrt{r^2 - b_1^2} + \beta \left(\frac{\beta \sqrt{r^2 - b_1^2} + r}{k} \right)}{\frac{\beta \sqrt{r^2 - b_1^2} + r}{k}} = \frac{\sqrt{r^2 - b_1^2} + \beta^2}{\beta \sqrt{r^2 - b_1^2} + r}$$

Introducing the parameter

$$Z = \sin \alpha_1^1 = \sqrt{1 - \left(\frac{b_1}{r}\right)^2} \quad \text{we have} \qquad (2)$$

$$\sin \alpha_1 = \frac{Z + \beta}{\beta Z + 1} \qquad (3)$$

hence $Z\,\beta\,\mathrm{Sin}\,\alpha_1 + \mathrm{Sin}\,\alpha_1 - Z - \beta = 0$

$$\text{and } Z = \frac{\beta - \mathrm{Sin}\,\alpha_1}{\beta\,\mathrm{Sin}\,\alpha_1 - 1} \tag{4}$$

We will now invastigate the mótion of ray in a medium and let us designate the velocity of light at the resting medium by q^1 and in the moving medium by q. Let us calculate tg α_2 (fig. 2) tg $\alpha_2 = \frac{a_2 + S_2}{b_2}$; $S_2 = vt$ time is designate by t and

$$\mathrm{tg}\,\alpha_2 = \frac{a_2 + vt}{b_2} \tag{5}$$

Here t is a time respecting to the observer who is resting together with a ether. For the observer who is moving together with a medium one part OA_2 of way of the ray will be made during the time $t^1 = \frac{r}{q^1}$ out of principle of Lorenz's formula of the transformation

$$t = \frac{t^1 + \frac{v}{c^2}x^1}{k},$$

where $x^1 = \sqrt{r^2 - b_2^2}$ is the coordinate of point A_2^1 (fig. 1).

From the equation of elipse

$$\frac{b_2^2}{r^2} + \frac{a_2^2}{k^2 r^2} = 1\,;\quad a_2 = k\sqrt{r^2 - b_2^2}$$

Substituting b_2 and t in the expression (5) and designating

$$\frac{v}{q^1} = \beta_1\,;\ \text{we obtain } \mathrm{tg}\,\alpha_2 = \frac{k^2\sqrt{r^2 - b_2^2} + \left(\frac{r}{q^1} + \frac{v}{c^2}\sqrt{r^2 - b_2^2}\right)}{k\,b_2} =$$

$$= \frac{k^2\sqrt{r^2 - b_2^2} + \beta_1 r + \beta^2\sqrt{r^2 - b_2^2}}{k\,b_2}$$

$$\text{and simplifying farther we have } \mathrm{tg}\,\alpha_2 = \frac{\sqrt{r^2 - b_2^2} + \beta_1 r}{k\,b_2} \tag{6}$$

The law of refraction in the resting sistem gives

$$\frac{a_1^1}{a_2^1} = n = \frac{\sqrt{r^2 - b_1^2}}{\sqrt{r^2 - b_2^2}}$$

$$\text{hence } \sqrt{1 - \frac{b_2^2}{r^2}} = \frac{\sqrt{1 - \left(\frac{b_1}{r}\right)^2}}{n}$$

and for as much the equality (2) we shall have

$$\sqrt{1 - \left(\frac{b_2}{r}\right)^2} = \frac{z}{n}\,;\ \text{and } \frac{b_2}{r} = \sqrt{1 - \frac{z^2}{n^2}}$$

Substituting in (6) we got finally

$$\operatorname{tg} \alpha_2 = \frac{\frac{z}{n} + \beta_1}{k\sqrt{1 - \frac{z^2}{n^2}}}, \quad \operatorname{tg} \alpha_2 = \frac{z + \beta_1 n}{k\sqrt{n^2 - z^2}} \qquad (7)$$

Let us consider now a concrete case $\alpha_1 = 0$ that is, the ray falls perpendicular to a medium and direction of its motion. For a medium that is at rest we should have $\alpha_2 = 0$; in case of a relative motion of a medium we shall have a value α_2 different from a zero, and a value α_2 can be calculated from a formulae (4) and (7).

The formula (4) gives $Z = -\beta$; substituting z in (7) we get

$$\operatorname{tg} \alpha_2 = \frac{-\beta + \beta_1 n}{k\sqrt{n^2 - \beta^2}}$$

and perceiving that $\beta_1 = \frac{v}{c}\frac{c}{q^1} = \beta n$; $\beta = \frac{\beta_1}{n}$

after a substitude

$$\operatorname{tg} \alpha_2 = \frac{-\frac{\beta_1}{n} + \beta_1 n}{k\sqrt{n^2 - \beta^2}} = \frac{\beta_1 n\left(1 - \frac{1}{n^2}\right)}{n\sqrt{1 - \beta^2}\sqrt{1 - \frac{\beta^2}{n^2}}} \qquad (8)$$

Developing the multiplicators with the redicals in a series about an increasing degrees β we have

$$\frac{1}{\sqrt{1 - \beta^2}} = 1 + \frac{1}{2}\beta^2 + \frac{1}{2}\frac{3}{4}\beta^4 + \ldots\ldots$$

$$\frac{1}{\sqrt{1 - \frac{\beta^2}{n^2}}} = 1 + \frac{1}{2}\frac{\beta^2}{n^2} + \frac{1}{2}\frac{3}{4}\frac{\beta^4}{n^4} + \ldots\ldots$$

Substituting in (8) and multiplying we find

$$\operatorname{tg} \alpha_2 = \beta_1\left(1 - \frac{1}{n^2}\right)\left[1 + \frac{1}{2}\beta^2\left(1 + \frac{1}{n^2} +\right)\ldots\ldots\right]$$

Opening the parenthesis and throwing the members with β^3 and higher degrees β we shall have finally

$$\operatorname{tg} \alpha_2 = \frac{v\left(1 - \frac{1}{n^2}\right)}{q^1} \ldots\ldots \qquad (9)$$

that is, the light ray deviates from primary direction in case of the relative motion of a medium.

It is interesting to notice that in the expression of the tangent of angel of the deviation comes in Fresnel's coefficient and that the angel of deviation of the light ray can be so constructed as if the velocity of light had been

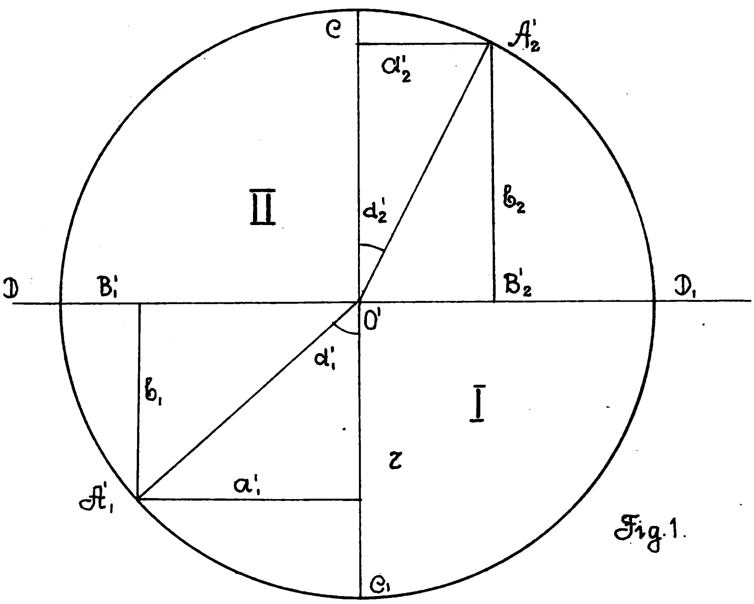
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Fig. 1.

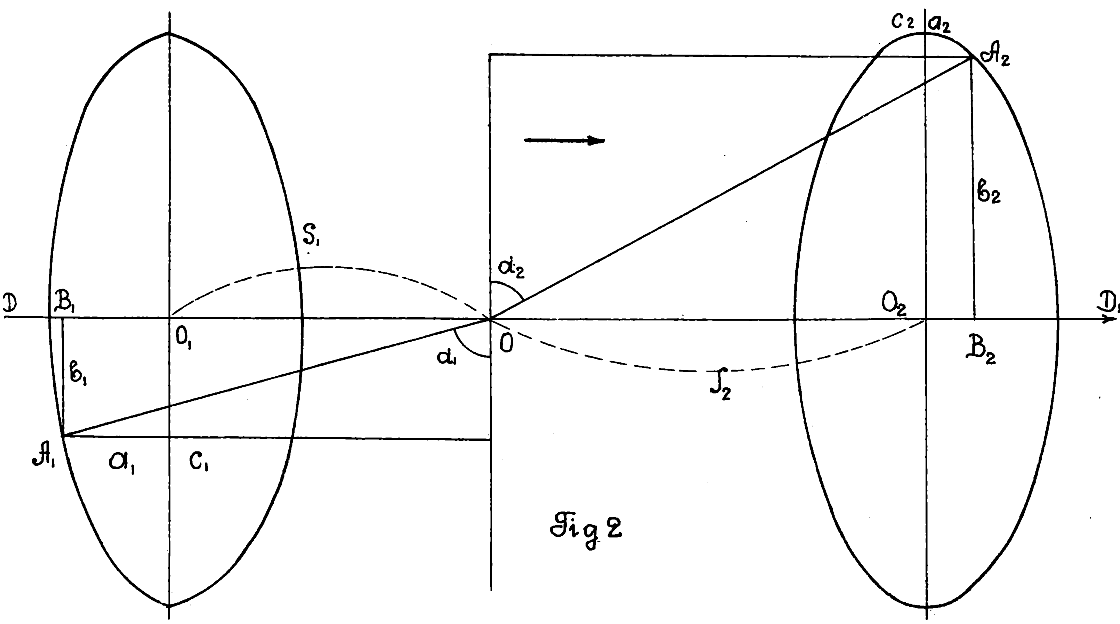
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Fig 2